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Edited for the Council by Capt. J. LAURENCE PRITCHARD, Hon. Fellow

CONTENTS FOR NOVEMBER, 1943

Monthly Notices

Some Notes on the Shear Centre of Thin-Walled Open Sections - *Haas*

Airscrew Blade Stress Due to Periodic Displacement—A Contribution to Cascade Theory - - - - *Lockwood Taylor*

Reviews

Abstracts from the Scientific and Technical Press

Titles and References of Articles and Papers

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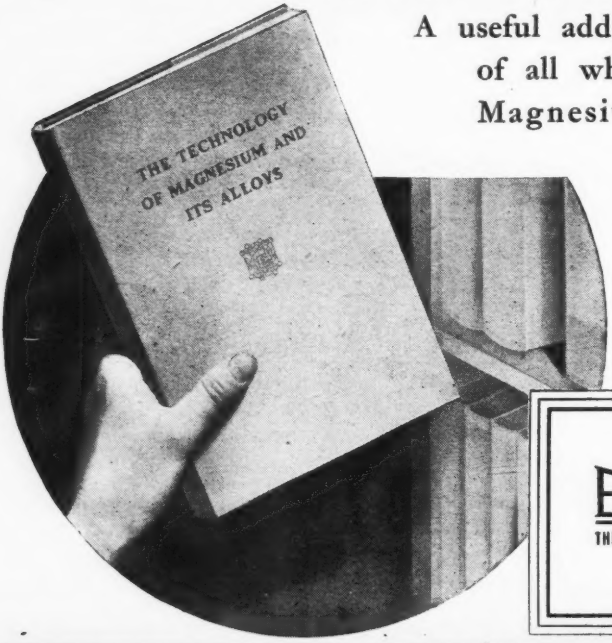

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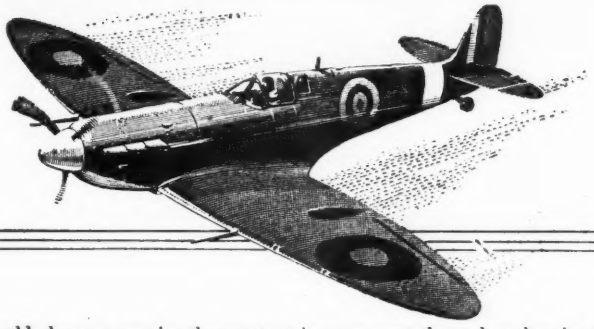

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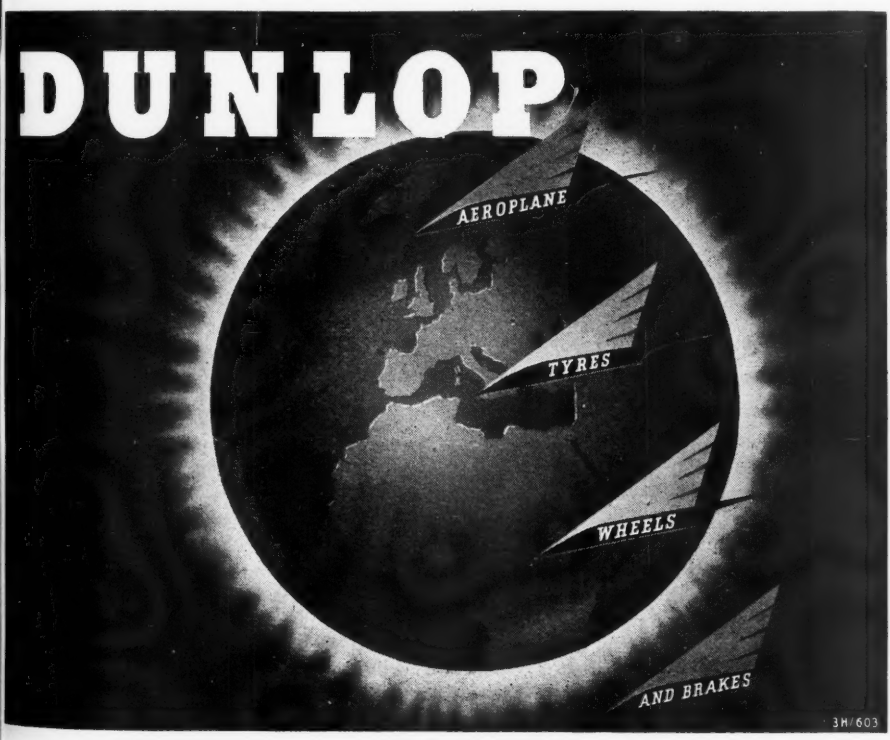

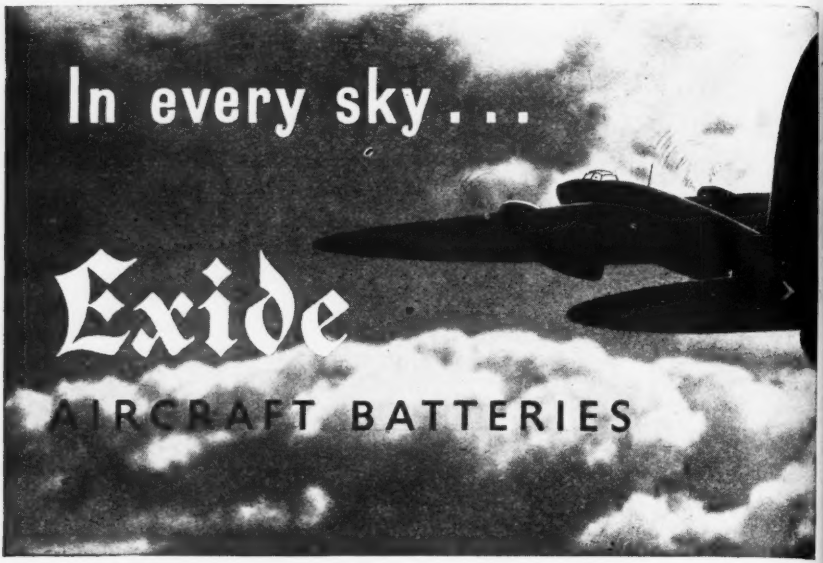

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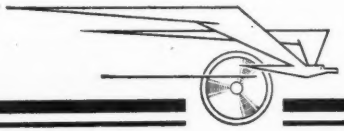

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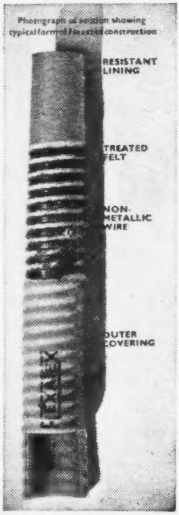

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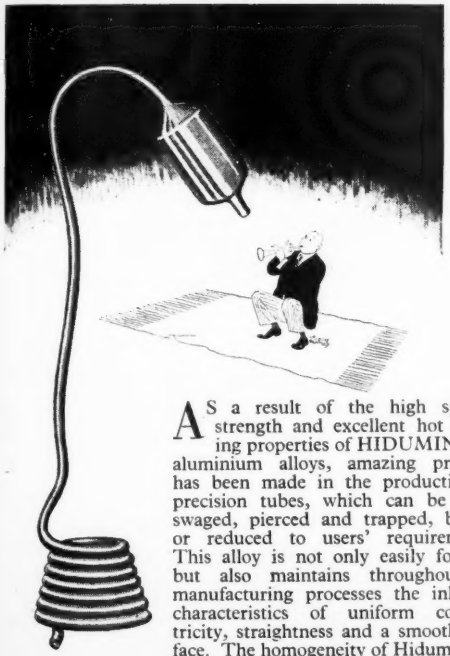

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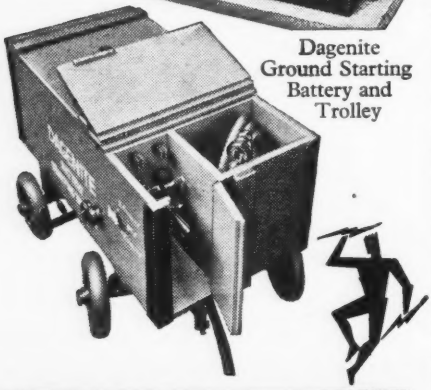

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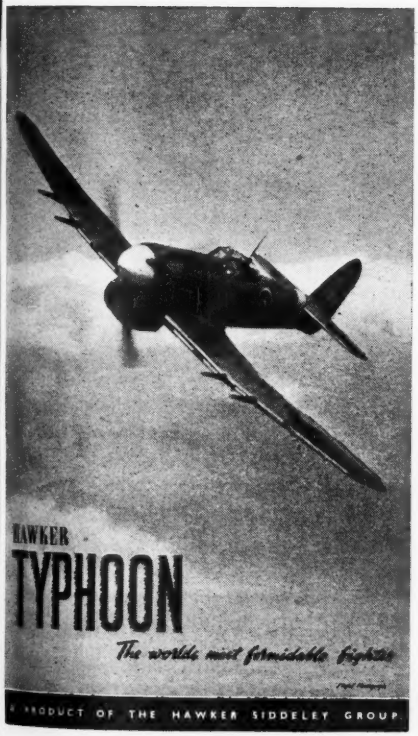

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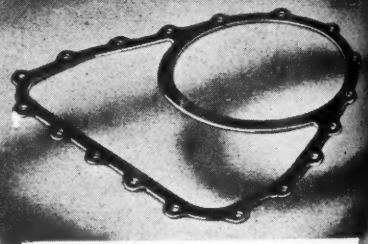

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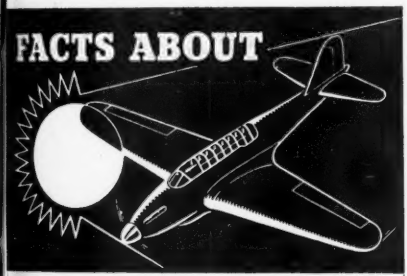

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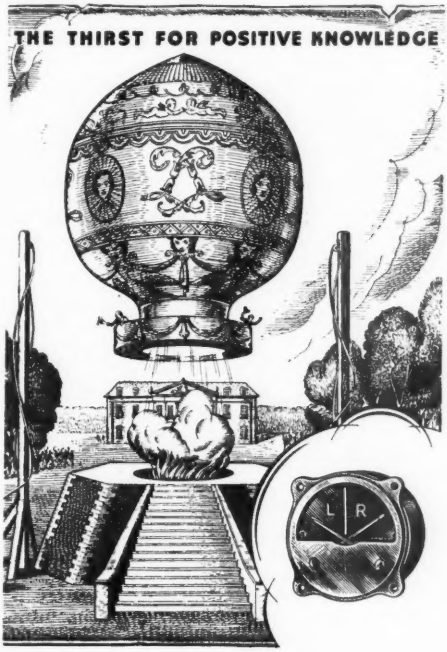

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CONTENTS FOR NOVEMBER, 1943

	PAGE
MONTHLY NOTICES	379
SOME NOTES ON THE SHEAR CENTRE OF THIN-WALLED OPEN SECTIONS, BY T. HAAS ...	383
AIRSCREW BLADE STRESS DUE TO PERIODIC DISPLACEMENT	390
A CONTRIBUTION TO CASCADE THEORY, BY J. LOCKWOOD TAYLOR	397
REVIEWS	401
ABSTRACTS FROM THE SCIENTIFIC AND TECHNICAL PRESS	*609*
TITLES AND REFERENCES OF ARTICLES AND PAPERS	*626*

THE JOURNAL OF THE ROYAL AERONAUTICAL SOCIETY *was founded in* 1897 *in succession to the* ANNUAL REPORTS.

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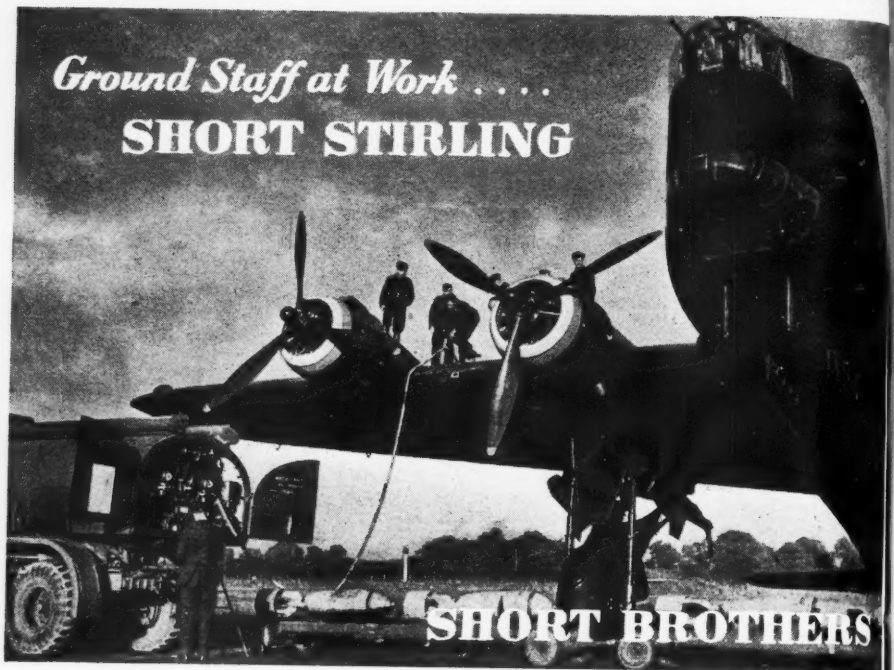

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NOVEMBER, 1943.

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Members are reminded that their 1944 subscriptions will become due on 1st January, 1944.

The Council are very grateful to all those members who have offered to forego their Journals for the duration of the war. The same reductions will be made to the 1944 subscriptions of these members as applied to the 1943 subscriptions. Subscriptions of Fellows, Associate Fellows, Graduates, Companions and Founder Members, who have given up their Journals will therefore be reduced by 10s. 6d. Associates will receive the reduction already laid down in the Rules and the low subscription at present paid by Students does not permit any reduction to be made.

The following will be the rates for 1944 therefore:—

	Home receiving Journal.	Home not receiving Journal.	Abroad receiving Journal.	Abroad not receiving Journal.
	£ s. d.	£ s. d.	£ s. d.	£ s. d.
Founder Members	2 2 0	1 11 6	2 2 0	1 11 6
Fellows	4 4 0	3 13 6	3 3 0	2 12 6
Associate Fellows	3 3 0	2 12 6	2 2 0	1 11 6
Associates	2 2 0	1 1 0	2 2 0	1 1 0
Graduates (age 21-25) ...	2 2 0	1 11 6	2 2 0	1 11 6
,, (age 26-28) ...	2 12 6	2 2 0	2 12 6	2 2 0
Students	1 1 0	1 1 0	1 1 0	1 1 0
Companions	2 2 0	1 11 6	2 2 0	1 11 6

Library Opening.

The Library is normally open on Mondays-Fridays inclusive from 9.30 a.m.—5.0 p.m. and Saturdays 9.30 a.m.—12.30 p.m. In addition it will also be open on Saturday afternoons from 2.0 p.m.—5.0 p.m. until December 1st in order to ascertain whether the numbers using the Library on Saturday afternoons justifies the continuance of the arrangement.

Society of British Aircraft Constructors' Scholarships.

The Scholarship Selection Committee of the Society of British Aircraft Constructors and the Royal Aeronautical Society have made awards of Society of British Aircraft Constructors' Scholarships to the following:—

James Roy Combley ... 18, Bridge Street, Leighton Buzzard. Educated at Leighton Buzzard Junior Boys' School; Cedars School, Bedford; and de Havilland Technical School.

Frederick Stanley Newson	133, New Bedford Road (South), Stretley, Luton, Bedfordshire. Educated at Luton Modern School.
Albert Seaman	11, Stafford Gardens, Croydon, Surrey. Educated at Cooper Road Elementary School; Archbishop Tenison's Central School.

Collection of Aeronautical Medals.

Thanks to the generosity of an anonymous donor, the Royal Aeronautical Society has recently acquired a remarkable collection of Aeronautical medals, over 300 in number, dating from 1714 to 1941. The medals are a veritable history of aeronautics, struck on the occasion of some aeronautical event.

Among the many medals which have been acquired it is possible to pick out only a few to give some idea of their range.

Three of the early medals were struck in 1783 to commemorate the first ascent of the Montgolfier Brothers in a hot air balloon; two others show the first ascent of Lunardi from London in 1784; another the first Dutch aeronaut to ascend, from Amsterdam, in 1806. James Sadler, the first Englishman to make an ascent, is commemorated in a medal dated 1811. A silver medal of 1851 shows the ascent of three balloonists each on a horse! There are many examples of the Siege of Paris medals of 1870, Giffard's ascents and others. A bronze medal of 1890 shows the Andrée polar balloon, and several silver ones of 1901 Santos Dumont's dirigible.

Coming to later days, there is a medal of 1909 commemorating the award of the Congress medal to the brothers Wright; medals of various Zeppelins, Parseval, the R.101, and other dirigibles; medals commemorating Bleriot's cross-channel flights, 1909; the flights of Paulhan (London-Manchester, 1910); Chavez (crossing the Alps, 1910); Pegoud (the first to loop the loop); Lindbergh (Trans-Atlantic flight, 1927); Bert Hinkler (London to Australia, 16 days, 1928); Kohl, Hunefeld and Fitzmaurice (Ireland to America, first east-west flight, 1928); Mussolini—Chief Airman, 6th year of Facism, 1928-9; Dornier D.O.X. Flying Boat, then largest in the world, 1929; Professor Picard and Dr. Kipfer, first Stratosphere ascent, 1931; Balbo's Armada of 24 seaplanes—Italy to America and back, 1933; and Amelia Earhart (1941).

Election of Members.

Associate Fellows.—John William Blinkhorn, Walter Stewart Brown, Thomas Travers Newton Coleridge, Tadeusz Czaykowski, Wilton William Waveney Downing, Lionel Harvey Elford, George William Hoggan Gardner, Hugh Tempello Grigg (from Graduate), Geoffrey Warren Hall, Kenneth Walter Hetzel (from Graduate), Frank Holoubek, Piotr Kubicki, Otto Jens Marstrand, Edward Richard Major, William James Muskett, Stanislaw Pawel Prauss, Arthur Francis Varney, Thomas Horsfall Ward, Murry Pearce White (from Student), Herbert Arthur Wise.

Associates.—Lawrence Bailey, John Walter Beak, William Henry Benbow, Reginald Stanley Bevan, Eric Thomas Barnard, Frederick William Robert Bird, James Cosgrove Carmichael, Francis John Cox, Roy Derek Cullum (from Student), Frank Ernest Davis, James Robert Duncan, Thomas Gillett, Leslie Herbert Hayward, Charles Albert Hornsby, Dudley Mortimer Hudson, Henry John Hyde, Robert Kego, Frank Midford-Millership (from Companion), Gerald Albert Frank Owen, John William Roach, James Martin Robins, Frederick Charles George Sawyers, Oliver Stafford Smart, Horace Smith, Robert George Henry Stone, Jack Storey, Trevor Hetherington Tarr (from Student), Raymond Ernest Wilby, Charles Harold Williams, Ernest William Leonard Wills.

Graduates.—Ronald Oliver Bater, Paul Cadman, Frederick Clay, Arthur Trevena James Crew, Maxwell John Edmonds, Gerald Gordon Fowler, Charles Esson Findlay, Gordon Stanley Gee, William Hampton (from Student), Edward James Jones, William Taylor Kirkby (from Student), Alan Norman Knowles, Kelvin Oliver Lockyear, Clifford Douglas Peters, Lewis Roy Seeling, Yuan Shen, George Sim, Charles Frederick Sloper, Arthur William James Upton, Lucien Sidney Wigdortchik, William Gerald Webber, Dennis Wilde.

Students.—Gordon Leyland Anderton, Guy Birch Brushfield, Basil Dunstan Brown, Alan Clegg, Geoffrey St. Quintin Crockett, Fredrick John Ernest Davison, Reginald George Evans, Leslie Fowler, Arthur Charles Frye, Derek Roger Garraway, Charles Phillip Gilbert, John Anthony Edward Goss, Michael John Gwinnell, Peter Reginald James Harding, Francis Henry Harper, Kenneth David Harris, Ronald Leslie Edmund Henson, Henry Philip Yeulett Hitch, John Edward Holl, Jack Hollingum, Donald Russell Jones, Donald William Kemp, John Alexander McCarthy, John McCulloch, Derek Moncrieffe Mackay, Bruce Howard Macnay, Frederick Avann Mason, Derek Ernest Milnes, Kenneth William Newby, Frederick Derrick Oliver, Charles Colin Parsons, Antony Mervin Peduzzi, Hilary John Silvanus Pinsent, David William Robinson, Kenneth David Ross, Peter John Ross, Eric William Saben, David Patrick Lindsay Scallon, Hanus Karel Sladkus, Bryan Charles Smith, John Speechley, Stanley Frank Stapleton, John Raymond Stevens, Trevor Edwin Arthur Stevenson, Herbert Stanley Taylor, William Idris Thomas, R. E. Vale, Charles William Ward, Geoffrey Godsall Webster, Gordon Thomas White, Bernard Wild, Victor Willis, Peter William Wreford, Dennis Wright.

Companions.—Thomas Edward Seymour Bell, Battista Louis Gagliardi, Basil Saravanos.

Acknowledgments.

The Council wish to place on record their thanks to Mr. H. E. Brand (Companion Member) and to F. W. Atkinson (Associate) for the gift of Journals of the Society. Many of those presented by Mr. Atkinson are very scarce and some have been out of print for some time.

Additions to Library.

Pamphlets in italics, with location reference following in brackets.
Books marked * or ** may not be taken out on loan.

A.a.298.—Aerodynamics of the Aeroplane. By W. L. Cowley. Thos. Nelson and Sons. 1943. 5/-. (Series: Nelson's Aeroscience Manuals.)

D.b.194.—*Post-War Air Transport. Air League of the British Empire.* 1943. (PD.1/24.)

D.b.195.—*Our Airlines—Our Lifelines. By Bruce Elliott. W. H. Allen and Co.* 1943. 9*d.* (PD.1/25.)

E.d.20.—New Methods for Sheet Metal Work. By W. Cookson. Technical Press, Ltd. 1941.

E.f.101.—Aircraft Inspection. By Ernest E. Wissman. McGraw Hill Book Co. 1942. 18/-.

G.a.61.—*The Technical Cohesive Strength and other Mechanical Properties of Metals at Low Temperatures. By D. J. McAdam and R. W. Mebs. (Preprint of Paper read at 46th Meeting of the American Society for Testing Materials, June,* 1943.) (PG.1/20.)

G.a.62.—*An Investigation of the Technical Cohesive Strength of Metals. By D. J. McAdam and R. W. Mebs. (Tech. Publication No. 1,615 of the American Institute of Mining and Metallurgical Engineers.)* 1943. (PG.1/21.)

*G.b.23.—Non-Corrodible Steels Data Book. (Revised Edition.) Firth-Vickers Stainless Steels, Ltd. 1943. (Loose-leaf folder.)

*G.c.16.—Manual on Presentation of Data. (With Supplements A and B.) American Society for Testing Materials. 1943. 85 cents. (To members, 65 cents.)

J.g.196.—Cloud Reading for Pilots. By A. C. Douglas. John Murray. 1943. 10/-.

M.b.50—Basic Radio. By C. L. Boltz. Thos. Nelson and Sons. 1943. 5/-. (Series: Nelson's Aeroscience Manuals.)

Q.b.71.—National Central Library: 26th and 27th Annual Reports (1941-42 and 1942-43). (In one volume.) 1943.

**R.b.246.—A Narrative of the Two Aerial Voyages of Doctor Jeffries with Mons. Blanchard, with Meteorological Observations and Remarks. By Dr. Jeffries. Privately printed, 1786.

S.e.69.—The Conquest of North Africa. Burke Publishing Co. 1943. 2/-.

T.a.105.—Knight of Germany (Oswald Boelcke). By Johannes Werner. (English translation by Claud Sykes.) John Hamilton. 1933.

*UU.c.—*National Advisory Committee for Aeronautics (U.S.A.): Technical Memoranda*:—

No. 1,047. Heat Transfer through Turbulent Friction Layers. By H. Reichardt. (From Z.f.a.M. and M., Dec., 1940.)

No. 1,048. Periodic Heat Transfer at Small Pressure Fluctuations. By H. Pfriem. (From: F.G.I., March-April, 1940.)

*WB.17/15.—The Nickel Bulletin, Vol. 15, 1942. Mond Nickel Co. 1943.

*WB.21.—Journal and Proceedings of the Royal Society of New South Wales. Vol. LXXVI, Part III (1942).

J. LAURENCE PRITCHARD, *Secretary and Editor.*

SOME NOTES ON THE SHEAR CENTRE OF THIN-WALLED OPEN SECTIONS.

By T. Haas, A.F.R.Ae.S.

Introduction.

Advanced textbooks of strength of materials dealing with the shear centre give examples of the simplest sections only (*e.g.*, Ref. 1, Art. 8). Numerous articles in the technical press are devoted to the subject, and the method to find the shear centre of any open section is now well established (Ref. 2). However, the number of readily usable solutions for sections common in practice, is small. Consideration in this paper is given to a more complicated section, fairly common in practice, and a formulæ is derived for the location of its shear centre. The section is a lipped channel, symmetrical about one axis, and the paper is followed by a diagram that enables the shear centre position for various ratios of lip to leg and leg to web to be read off.

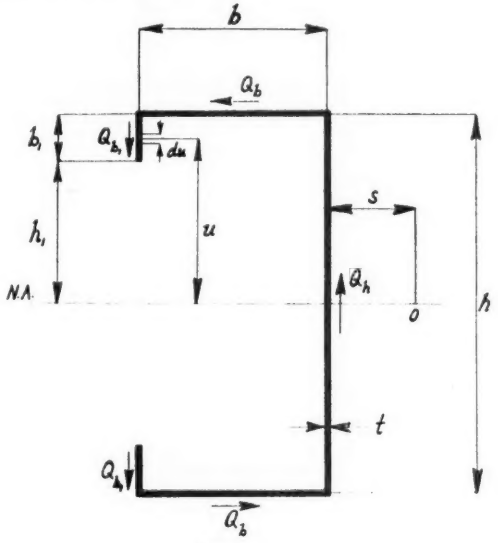

Fig. 1.

Dimensions of lipped channel section and internal shear forces.

A short critical review of available literature on the determination of the shear centre has been included to ease the designer's work when looking for directly applicable formulæ.

Definition and Determination of Shear Centre.

In the theory of bending, the shear centre or flexural centre is defined as that point through which the applied transverse load must pass if the bending be

unaccompanied by torsion. By applying the reciprocal theorem it can at once be seen that under pure torsion the section will rotate about the same point—the centre of twist. Thus the centre of twist is identical with the shear centre of the section (Ref. 3).

In order to determine the location of the shear centre use is made of the above definition in stating alternately that the resultant of the internal shear forces must pass through the shear centre to produce bending without torsion.

Applying this to the section under consideration (Fig. 1), the sum of all moments about the shear centre O must be zero, $Q_h s - Q_b h - 2Q_{b_1}(h+s)=0$, or in other words the resultant torque of all the shear stresses vanishes.

Therefore the distance of the shear centre from the web is

$$s=\frac{(2Q_{b_1}h+Q_b h)}{(Q_h-2Q_{b_1})} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The axis of symmetry of the section coincides with the neutral axis as the applied transverse load is parallel to the web. To evaluate Eq. (1) it is necessary to consider the distribution of shear stresses over the section in simple bending, in which all parts of the section are assumed to be fully effective. The distribution of shear stresses over the thickness t is assumed to be uniform, a requirement which can be satisfied by using thin-walled sections. The possibility of the instability of the walls, however, is excluded.

To begin with, the resultant Q_{b_1} of the shear stresses distributed over the lip is calculated,

$$Q_{b_1}=\frac{Q}{I_{NA}}\int_{h1}^{h/2} S\,du \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where Q is the applied transverse load, and I_{NA} is the moment of inertia of the section about the neutral axis. The static moment of the lip,

$$S=t\int_{h1}^{u} u\,du=t\left(\frac{u^2}{2}-\frac{h_1^2}{2}\right) \quad . \quad . \quad . \quad . \quad (3)$$

Substituting (3) in (2) and integrating, we have

$$Q_{b_1}=\left(\frac{Q}{I_{NA}}\right)t\left[\frac{u^3}{6}-\frac{uh_1^2}{2}\right]_{h1}^{h/2}=\frac{Q}{I_{NA}}\,tb_1^2\left(\tfrac{1}{4}h-\tfrac{1}{3}b_1\right) \quad . \quad . \quad (4)$$

It is convenient to express Q_{b_1} in terms of the maximum shear stress τ_1 at corner A of the lip over which the shear stresses are distributed parabolically (Fig. 2). Thus, the bending shear stress

$$\tau_1=\frac{S_{b_1}Q}{I_{NA}t}=\left(\frac{Q}{I_{NA}}\right)\frac{1}{2}\left(\frac{h^2}{4}-h_1^2\right) \quad . \quad . \quad . \quad . \quad (5)$$

whereby

$$S_{b_1}=t\int_{h1}^{h/2} u\,du=(t/2)\left(h^2/4-h_1^2\right). \quad . \quad . \quad . \quad . \quad (6)$$

Therefore from Eq. (4) and Eq. (5)

$$Q_{b_1}=\frac{\left\{(Q/I_{NA})\,tb_1^2\left(\frac{1}{4}h-\frac{1}{3}b_1\right)\right\}}{\left\{(Q/I_{NA})\,\frac{1}{2}\left(h^2/4-h_1^2\right)\right\}}\cdot\tau_1$$

and finally, with $h_1=(h/2)-b_1$,

$$Q_{b_1}=\tau_1 tb_1\frac{\left(\frac{1}{2}-\frac{2}{3}b_1/h\right)}{(1-b_1/h)} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Proceeding further in the same way, the distribution of shear stresses over the whole section has been determined; this is shown in Fig. 2. The shear stresses τ_0 at corner B and τ_3 at point C of the section can be expressed in terms

of τ_1, and thus the resulting shear forces Q_h and Q_h in the legs and the web respectively, can be written down.

Thus, $\tau_0 = \tau_1 + (QS_h/I_{NA}t)$, where the static moment of the leg, $S_h = (bt)\,h/2$. Then

$$\tau_0 = \tau_1 \left\{ 1 + \left(\frac{Q}{\tau_1}\right)\left(\frac{bt\,h/2}{I_{NA}t}\right) \right\}$$

By substituting Eq. (5), and for $h_1 = (h/2) - b_1$

$$\tau_0 = \tau_1 \left\{ 1 + \left(\frac{b/b_1}{(1 - b_1/h)}\right) \right\}$$

and

$$Q_h = \tfrac{1}{2}(\tau_0 + \tau_1)\,bt = \tfrac{1}{2}\tau_1 bt \left\{ 2 + \frac{b/b_1}{(1 - b_1/h)} \right\} \quad . \quad . \quad . \quad (8)$$

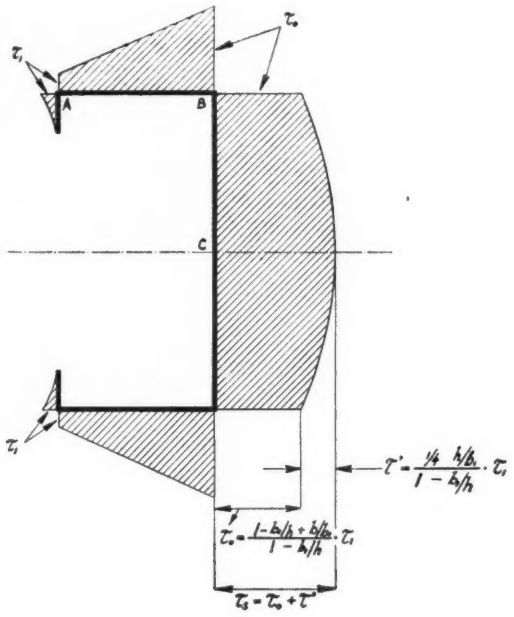

FIG. 2.

Distribution of shear stress in walls of a lipped channel section.

Similarly, for the web (see Fig. 2),

$$\tau_s = \tau_0 + \frac{QS_h}{I_{NA}t}$$

the static moment of the web being

$$S_h = (h^2/8)\,t.$$

Thus it can be written,

$$\tau_s = \tau_1 \left(\frac{\tau_0}{\tau_1} + \frac{Q\,h^2/8}{\tau_1 I_{NA}} \right)$$

and on substituting Eq. (5), and for $\tau_0/\tau_1 = 1 + \{(b - b_1)/(1 - b_1/h)\}$ as before,

$$\tau_s = \tau_1 \left\{ 1 + \frac{b/b_1}{(1 - b_1/h)} + \frac{\frac{1}{4}h/b_1}{(1 - b_1/h)} \right\}$$

As $\tau_a + \tau' = \tau_1$,

$$\tau' = \tau_1 \frac{\frac{1}{4}(h/b_1)}{(1 - b_1/h)}$$

The resultant shear force in the web is

$$Q_h = \tau_a ht + \tfrac{2}{3}\tau' ht = \tau_1 ht \left\{ 1 + \frac{(b/b_1 + \frac{1}{6}h/b_1)}{(1 - b_1/h)} \right\} \quad . \quad . \quad . \quad (9)$$

Equations (7), (8) and (9) substituted into Eq. (1) yield

$$s = b \frac{\{ 1 + \frac{1}{2}b/b_1 - \frac{4}{3}(b_1/h)^2 \}}{\{ 1 + b/b_1 + \frac{1}{6}h/b_1 - 2(b_1/h)(1 - \frac{2}{3}b_1/h) \}} \quad . \quad . \quad . \quad (10)$$

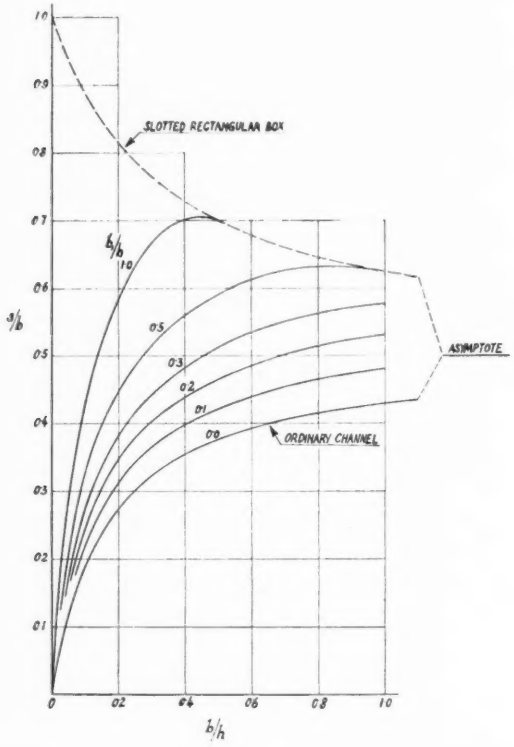

FIG. 3.

Shear centre location for lipped channel sections (Eq. 10).

This equation is plotted in non-dimensional form in Fig. 3 for various values of the ratio b_1/b as parameter. The lower limiting curve for $b_1 = 0$ represents the ordinary channel section and the values for $s = b/(2 + \frac{1}{3}h/b)$ are in agreement with the results given in Refs. 2 and 4.

The upper limiting curve shown dotted is the locus for all values of $b_1 = h/2$, *i.e.*, the cross section of a slotted rectangular box.

In Equation (10) the wall thickness t which has been taken as uniform cancels out. However, for different thicknesses t_h, t_b and t_{b_1} of web, legs and lips

respectively t in Equations (7), (8) and (9) should be replaced by the appropriate thicknesses to obtain the correct shear centre location on substitution into Eq. (1).

Another section to be derived from the above equations suggests itself immediately in the form of the wing channel, Fig. 4. Here the resultant shear forces Q_{b_1} in the lips are of equal direction with Q_h the shear force in the web, so that for this case the sign of Q_{b_1} in Eq. (1) should be reversed.

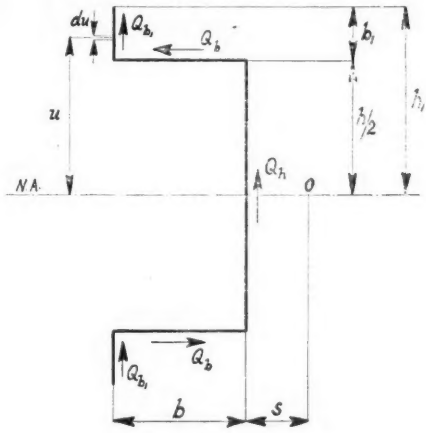

FIG. 4.
Wing channel section.

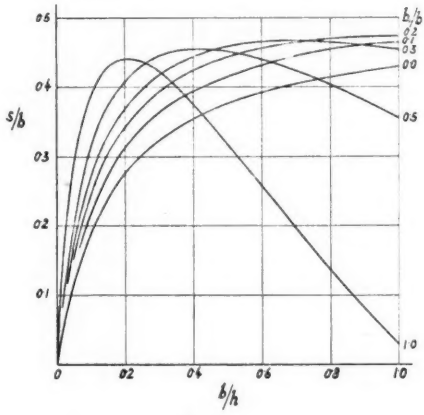

FIG. 5.
Shear centre location for wing channel sections (Eq. 11).

Likewise, the static moment of the wing increases with decreasing u (Fig. 4). Therefore, Eq. (3) holds good with the right hand side negative, and putting $h_1=(h/2)+b_1$.

The resulting expression for the shear centre distance from the web is then

$$s=b\frac{\{1+\frac{1}{2}b/b_1-\frac{4}{3}(b_1/h)^2\}}{\{1+b/b_1+\frac{1}{6}h/b_1+2(b_1/h)(1+\frac{2}{3}b_1/h)\}} \quad . \quad . \quad (11)$$

For a few values of b_1/b, curves for this section have been plotted too, and are shown in Fig. 5.

The diagrams in Figs. 3 and 5 could of course be plotted in terms of a single parameter, say, h, but would not be so instructive.

Review of Literature.

No attempt is made in the following short survey to collect an exhaustive bibliography as the references quoted at the end of the article contain further references, which taken altogether give a fairly complete picture of what has been written on the subject. It is rather the problem of torsion and warping which brings out the importance of the shear centre to the structural engineer (Ref. 1, Art. 53). Hoff presented a method for the calculation of the warping constant (Ref. 3, Art. 7) for which the knowledge of the shear centre location is required. Art. 9 of Ref. 3 contains also formulæ for the shear centre of common thin-walled sections, including the lipped equal angle section (Fig. A6, c).

For sections that have no axis of symmetry, the process described of finding the shear centre would have to be carried out twice, *i.e.*, the section loaded in one direction, the corresponding resultant shear force determined, and the same repeated for another load direction. The intersection of these two resultant shear forces is then the shear centre. For asymmetrical channel sections Schwalbe (Ref. 5) presented a usable equation for locating the shear centre.

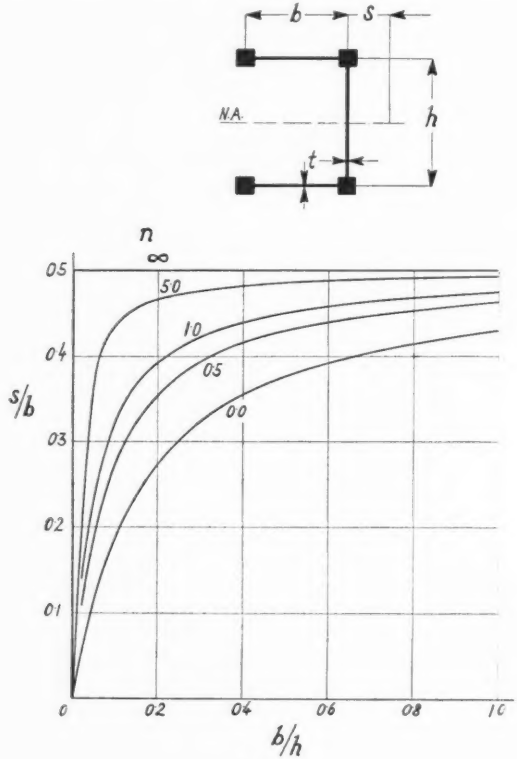

Fig. 6.

Shear centre location for channel section with concentrated members.

It should be pointed out in this connection that all sections treated in Refs. 1, 2, 3 and 5 are thin-walled, whilst Wagner (Ref. 4) and Kuhn (Ref. 6) include sections having concentrated members, whereby the thin-walled parts of the section are not assumed to be effective in bending. Thus there is no bending shear stress over the thin parts, and therefore the shear centre distance is numerically different from that of thin sections which have no concentrated members.

To illustrate the influence of the various assumptions let us consider the ordinary channel section with concentrated members. The area of the thin part is $t(2b+h)$; that of each of the four concentrated members be nbt. Then, the distance of the shear centre, as easily found is

$$s = \frac{\{ b(1+2n) \}}{\{ 2+\frac{1}{3}(h/b)+4n \}}$$

This is plotted in Fig. 6 with n as parameter, showing that for the limiting values $n=0$ (no concentrated members) and $n=\infty$ (thin-walled parts ineffective in bending) the results are widely different.

The shear centre location of an open circular arc is given in Ref. 2; for elliptic arcs and arcs with varying curvature it seems that at present only by graphical integration can one arrive at a solution.

Taken on the merits of the problems connected with the shear centre it appears desirable that data on the shear centre location be included into sectional data generally given in engineering handbooks and standards.

References.

(1) S. Timoshenko, Strengths of Materials, Part II, Second Ed., 1941.
(2) D. Williams, Behaviour in Bending of Thin-walled Tubes and Channels. 1935, R. & M. 1669.
(3) N. J. Hoff, Stresses in Space Curved Rings Reinforcing the Edges of Cut-outs in Monocoque Fuselages. J. of R. Ae. S., February, 1943.
(4) H. Wagner, Einige Bemerkungen über Knickstäbe und Biegungsträger. Der Kennwert. Z.F.M., Vol. 19, 1928, pp. 241-247.
——— Remarks on Airplane Struts and Girders under Compressive and Bending Stresses—Index Values. N.A.C.A. T. Memorandum, No. 500, 1929.
(5) W. L. Schwalbe, The Centre of Torsion for Angle and Channel Sections. A.S.M.E. Transactions, Vol. 54, 1932; APM-54-11, pp. 125-130.
(6) P. Kuhn, Some Elementary Principles of Shell Stress Analysis with Notes on the Use of the Shear Centre. N.A.C.A. T.N. No. 691, 1939.

AIRSCREW BLADE STRESS DUE TO PERIODIC DISPLACEMENT.*

Report of the German Aircraft Research Station E.V. Berlin-Adlershof
Institute for Driving Mechanism.

(L.F.F., Vol. 18, 20/11/41, pp. 383-386.)

Translated by G. M. W. KNOTT.

Airscrew blade stresses caused by periodic displacement of the airscrew shaft and the possibility of their elimination by elastic connection of motor and airscrew are investigated.

ARRANGEMENT.

I. Introduction.
II. Stress due to airscrew shaft vibration about an axis perpendicular to it.
III. Stress due to vibrations of the screw shaft about two axes perpendicular to it.
IV. Calculation of additional springing between engine and airscrew.
V. Summary.
VI. Bibljography.

I. INTRODUCTION.

An effective means of protecting the airscrew from alternating bending moments is the all-round elastic connection of engine and airscrew. Up to the present, however, owing to constructional difficulties such a design fulfilling all requirements has not yet been produced. An ideal solution would be airscrew blades (1) universally jointed as in helicopters. For normal screws this end is not directly attainable owing to heavy load on the joint. Particularly in order to reduce the coupling of airscrew bending oscillations with the crankshaft rotational oscillations the torque rod and flexible drive have been evolved as partially elastic connections. Since, on the other hand, crankshaft vibrations have been successfully overcome, for example, by fitting centrifugal penduli and dampers, the danger of screw overload may be considered as eliminated in many cases, by blade vibrations which must maintain the equilibrium with a rotational vibration moment of the crankshaft.

Recently cases of airscrew breakages have occurred obviously occasioned by tilting movements of the whole engine. The screw motion as a whole can be upset by this in a displacement within the plane of the airscrew disc and in a falling motion about the screw centre of gravity. Whilst the former gives rise to mass forces or moments within the plane of the screw disc, the latter causes alternating centrifugal moments perpendicular thereto, both of which may give rise to natural bending oscillations of the blades. In America these phenomena have been the object of thorough research for a considerable time (5). The excitation of these vibrations is produced on the engine side by gas and mass forces or from the airscrew side by air forces, *e.g.*, the screw blades sweeping past flow obstacles. In what follows the two moments just referred to will be deduced from the engine and screw motion respectively.

* Published by permission of the Ministry of Aircraft Production, R.T.P.

II. STRESS DUE TO AIRSCREW SHAFT VIBRATION ABOUT AN AXIS PERPENDICULAR TO IT.

Symbols used in the subsequent computation process:—

ωt = screw blade angle of torsion about the axis of rotation.
ϕ = screw shaft angle of torsion about an axis perpendicular to it.
m = the blade mass.
s = centre of gravity distance from blade root.
$J = \int_{r_1}^{R} r^2 dm - r_1 \int_{r_1}^{R} dm$ with r_1 as reference radius for the blade root.

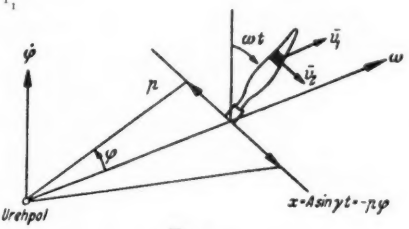

FIG. 1.

For deflection of the forces of inertia exerted on the blade by airscrew shaft displacement.
Drehpol = rotary pole.

For the calculation we regard (Fig. 1) appropriately a mass element of the screw blade and calculate the speed v thereof from the two rotational speeds ω and ϕ. Therefrom we get the acceleration through differentiation according to the time and thereby the force of inertia of the mass element. This integrated over the whole blade gives the bending moment acting on the blade root.

We then split the acceleration perpendicular to the plane of the screw disc into two parts

$$\frac{dv}{dt} = \frac{dv_1}{dt} + \frac{dv_2}{dt}$$

whereby

$$v_1 = r \sin \omega t \,.\, \dot{\phi}; \quad v_2 = r \cos \omega t \,.\, \omega;$$

from this it follows

$$\frac{dv_1}{dt} = r\,(\omega \dot{\phi} \cos \omega t + \ddot{\phi} \sin \omega t)$$

$$\frac{dv_2}{dt} = r \cos \omega t \,.\, \omega \,.\, \dot{\phi}; \text{ therefore } \frac{dv_2}{dt} = v_2 \frac{d\bar{\phi}}{dt}$$

Hence the bending moment becomes

$$M_\varphi = J\,(\ddot{\phi} \sin \omega t + 2\omega \dot{\phi} \cos \omega t).$$

For ϕ we can write $\phi = \Phi \sin \gamma t$ whereupon the above expression assumes the form

$$\begin{aligned} M_\varphi &= -J\gamma^2\Phi \,\{\, \sin \gamma t \sin \omega t - 2\,(\omega/\gamma) \cos \gamma t \cos \omega t \,\} \\ &= \tfrac{1}{2} J\gamma^2\Phi \,\{\, [\,1 + 2\omega/\gamma\,] \cos\,(\gamma+\omega)\,t \\ &\qquad - [\,1 - 2\omega/\gamma\,] \cos\,(\gamma-\omega)\,t \,\} \end{aligned}$$

Especially if $\gamma = \omega$ it follows

$$M_\varphi = \tfrac{1}{2} J\gamma^2\Phi \,\{\, 3 \cos 2\omega t + 1 \,\}.$$

To this bending moment perpendicular to the plane of the screw disc is superimposed one arising from the mass forces released by movement of the centre of gravity of the screw. The amplitude of this vibration is indicated

by $A=-p\,.\,\phi$ whereby p is the distance to the axis of rotation. Thus can be written $x=A\,.\,\sin\gamma t$. Hence the bending moment in the plane of the airscrew disc is

$$\begin{aligned} M_x &= -ms\gamma^2 A \sin\gamma t\cos\omega t \\ &= -\tfrac{1}{2}ms\gamma^2 A\,\{\sin(\gamma+\omega)\,t+\sin(\gamma-\omega)\,t\}\,. \end{aligned}$$

Particularly if $\gamma=\omega$ we have

$$M_x=-\tfrac{1}{2}ms\gamma^2 A\sin 2\omega t.$$

III. Stress Due to Vibrations of the Screw Shaft about Two Axes Perpendicular to it.

If the screw shaft furthermore swings about the second axis perpendicular to it with the deflection

$$\psi=\bar{\psi}\cos\gamma t,\quad B=p\,.\,\bar{\psi},\quad y=B\cos\gamma t,$$

so that the screw centre point describes in general an ellipse instead of a straight line the bending moment perpendicular to the plane of the airscrew disc becomes

$$\begin{aligned} M_z,\ \psi &= J\,\{\ \ddot{\psi}\sin\omega t+2\omega\dot{\psi}\cos\omega t+2\omega\dot{\psi}\sin\omega t-\ddot{\psi}\cos\omega t\ \} \\ M_z,\ \psi &= J\,\{\ -\gamma^2\,[\Phi\sin\gamma t\sin\omega t-\psi\cos\gamma t\cos\omega t] \\ &\qquad +2\omega\gamma\,[\Phi\cos\gamma t\cos\omega t-\psi\sin\gamma t\sin\omega t]\ \} \\ M_z,\ \psi &= J\,\{\ \tfrac{1}{2}(\Phi+\psi)\,[\gamma^2+2\omega\gamma]\cos(\gamma+\omega)\,t \\ &\qquad -\tfrac{1}{2}(\Phi-\psi)\,[\gamma^2-2\omega\gamma]\cos(\gamma-\omega)\,t\ \}\,. \end{aligned}$$

The mass moment is accordingly

$$\begin{aligned} M_x,\ y &= ms\,\{\ \ddot{x}\cos\omega t-\ddot{y}\sin\omega t\ \} \\ M_x,\ y &= -ms\gamma^2\,\{\ A\sin\gamma t\cos\omega t-B\cos\gamma t\sin\omega t\ \} \\ M_x,\ y &= -ms\gamma^2\,\{\ \tfrac{1}{2}(A-B)\sin(\gamma+\omega)\,t+\tfrac{1}{2}(A+B)\sin(\gamma-\omega)\,t\ \} \end{aligned}$$

In the special case of a rotary oscillation, *i.e.*, $\Phi=\psi$, $A=-B$ (precession in contrary motion) we have

$$\begin{aligned} M_z,\ \psi &= J\gamma^2\Phi\,\{\ [1+2\omega/\gamma]\cos(\gamma+\omega)\,t\ \} \\ M_x,\ y &= -ms\gamma^2 A\sin(\gamma+\omega)\,t. \end{aligned}$$

With precession in the same direction, *i.e.*, $\Phi=-\psi$ and $A=+B$ it follows

$$\begin{aligned} M_z,\ \psi &= -J\gamma^2\Phi\,\{\ [1-2\omega/\gamma]\cos(\gamma-\omega)\,t\ \} \\ M_x,\ y &= -ms\gamma^2 A\sin(\gamma-\omega)\,t. \end{aligned}$$

Accordingly the greatest values of the alternating bending moments due to rotary vibration are exactly twice as great as with a degree of freedom.

As long as the screw is carried by the driving shaft an all-round elastic fixing of the former is not possible as it would hang through under the influence of its own weight. The displacement prescribed by the screw shaft must consequently always be arranged to suit the screw. Yet an attachment is conceivable whereby at least the screw might not be compelled to comply to the full extent with the falling motion. As can be observed from the above expressions the two alternating moments are offset in both cases at 90° phase so that the momentum vector in the blade root rotates during a vibration period once in 360°. As the durability at least for isotropic materials only depends upon the extent of the alternating moments, and not upon them the question as to whether these are effective over one or over all directions of the cross section, an elimination of the centrifugal moment would only therefore be gained if they substantially exceeded the mass moments. As a comparison let us put down for the moment of inertia of the blade on account of its practically linear mass distribution $J=m\,(l^2/6)$ where l signifies the blade length and $s=l/3$. Thus in an unfavourable case we have

$$\begin{aligned} M_z,\ \psi &= J\gamma^2\Phi\,(1+2\omega/\gamma)=-(m/6)\,l^2\gamma^2\,(A/p)\,(1+2\omega/\gamma) \\ M_x,\ y &= -ms\gamma^2 A=-(m/3)\,l\gamma^2 A \\ \frac{M_z,\ \psi}{M_x,\ y} &= \frac{(1+2\omega/\gamma)\,l}{2p} \end{aligned}$$

This ratio is even greater than 1 for higher values of γ when it concerns radial engines with large reduction, therefore with large airscrews since then l is great in comparison to p. Especially in the very frequent case of the vibration being excited by an unbalance of the screw, *i.e.*, we have $\gamma=\omega$, this ratio can be very great.

$$\frac{M_{\varphi\,\mathrm{max}}}{M_{y,\,\mathrm{max}}}=\frac{1{\cdot}5\,l}{p}\sim 4.$$

Still other reasons meanwhile call for elimination of centrifugal moments. Summed over all blades they combine mainly in their effort on the screw shaft and seek to bend it, against which the mass moments reciprocally rise and only press the screw shaft more or less powerfully against its bearing. Finally this kind of stress appears to be unsuitable for wooden blades as regards the fixing in the steel sleeve, but this however can only be demonstrated after tests.

Above all it should be remembered that the bending moments calculated above, under a given screw displacement Φ, ψ or A, B represent the lowest values, consequently by increase of resonance much greater ones might actually accrue.

From the above considerations it thus appears that additional springing protecting the airscrew against alternating centrifugal moments, has its advantages provided it can be carried out with small expenditure of weight. Consequently in what follows let us consider questions after calculation of this additional springing and newly occurring resonance zones.

IV. Calculation of Additional Springing Between Engine and Airscrew.

In the calculation of springing we must proceed from the fact that no new critical revolutions may arise herefrom in the working revolution range and that moreover, with the centrifugal moment occurring in the turn the screw does not oppose the aircraft more than 10 per cent. Below we shall investigate whether both requirements can be simultaneously fulfilled.

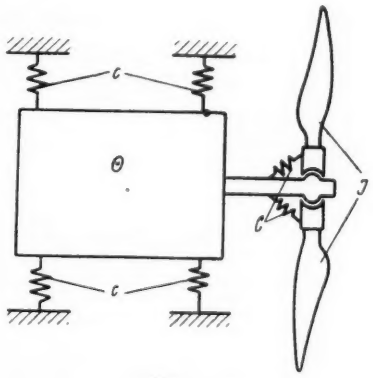

Fig. 2.

Schematic sketch of vibration system of engine with elastically connected screw.

Fig. 2 shows schematically the vibration system formed by elastic connection. The symbols employed are:—

Θ_1 = the moment of inertia of the engine plus screw mass about the transverse axis.

Θ_2 = the moment of inertia of the engine plus screw mass about the transverse axis.

c_1 = springing of the engine about the vertical axis.
c_2 = springing of the engine about the transverse axis.
C = springing of hub (equal on all axes).
J_p = the polar and
J = equatorial moment of inertia of the screw.
ω = screw angular velocity.
ϖ = aircraft angular velocity in the turn.
γ = vibration frequency.
ϕ and α = angular deflections of engine or screw about the vertical axis.
ψ and β = angular deflections of engine or screw at the transverse axis.

With these signs the equations of motion for the vibration system schematically drawn in Fig. 1 read:—

$$\left.\begin{array}{l} \text{I. } \Theta_1\ddot{\phi}+c_1\phi+C\,(\phi-\alpha)=0 \\ \text{II. } \Theta_2\ddot{\psi}+c_2\psi+C\,(\psi-\beta)=0 \\ \text{III. } J\ddot{\alpha}+C\,(\alpha-\phi)-J_p\omega\dot{\beta}=0 \\ \text{IV. } J\ddot{\beta}+C\,(\beta-\psi)+J_p\omega\dot{\alpha}=0 \end{array}\right\} \quad . \quad . \quad . \quad . \quad (1)$$

With the addition

$$\begin{array}{ll} \phi=\bar{\Phi}\sin\gamma t & \alpha=A\sin\gamma t \\ \psi=\bar{\psi}\cos\gamma t & \beta=B\cos\gamma t \end{array}$$

we have

$$\left.\begin{array}{l} \text{I. } (c_1+C-\Theta_1\gamma^2)\,\Phi=CA \\ \text{II. } (c_2+C-\Theta_2\gamma^2)\,\psi=CB \\ \text{III. } (C-J\gamma^2)\,A+J_p\omega\gamma B=C\Phi \\ \text{IV. } (C-J\gamma^2)\,B+J_p\omega\gamma A=C\psi \end{array}\right\} \quad . \quad . \quad . \quad . \quad (2)$$

If we multiply in (2) Eq. (o) and (II) with C and insert (III) and (IV) it follows

$$\begin{array}{l} \{\,c_1+C-\Theta_1\gamma^2\,\}\,\{\,(C-J\gamma^2)\,A+J_p\omega\gamma B\,\}=C^2A \\ \{\,c_2+C-\Theta_2\gamma^2\,\}\,\{\,(C-J\gamma^2)\,B+J_p\omega\gamma A\,\}=C^2B \end{array} \quad . \quad . \quad (3)$$

The four natural frequencies are then obtained from equation (8) for γ, which is obtained from (3) by elimination of A and B. For the sake of simplicity we write $c_1=c_2=c$ and $\Theta_1=\Theta_2=\Theta$ which is approximately correct for radial engines and obtained from (3) since then $A=\pm B$.

$$\gamma^4-\gamma^2\,\{\,C/J+(c+C)/\Theta\,\}+cC/J\Theta\pm 2\omega\gamma\,\{\,\gamma^2-(c+C)/C\,\}=0. \quad . \quad (4)$$

From this the four natural frequencies are to be calculated from the position of which the extent of C is to be determined. They should as far as possible lie so that the corresponding critical revolutions lie far beneath the range of working revolution. It is, however, unnecessary at the outset strictly to solve Eq. (4). It is certain that C will at least be so weak that the screw hardly accompanies the tipping motions of the engine, *i.e.*, the coupling between the pairs of equations I, II and III, IV in (1) is very slack. As in the case of a two-mass system, $K=\sqrt{\{\,C/(c+C)\,\}}$ applies to the coupling factor which should here be of the order of <0.3. Then again the couple frequencies appearing are not very different from the natural frequencies derived from the uncoupled equations. These are derived from (1) if in the first pair of equations α and β and in the second ϕ and ψ are put equal to o. In the simplified case of the uncoupled tipping vibrations of the engine at the vertical and transverse axes being equal to each other Eq. (4) can then be written

$$\{\,\gamma^2-(c+C)/\Theta\,\}\,\{\,\gamma^2\pm 2\omega\gamma-C/J\,\}=0 \quad . \quad . \quad . \quad (5)$$

Hence the natural frequencies are

$$\gamma_{1,2}=\sqrt{\{\,(c+C)/\Theta\,\}} \qquad \gamma_{3,4}=\sqrt{(C/J)}\,.\,\sqrt{\{\,v/(v\pm 2)\,\}}$$

whereby $v=\gamma/\omega$. If k_L is the source of excitation on the screw side and k_M on the engine side, in the first case $v=k_L$ and in the second case $v=k_M\,.\,i$ with i as reduction ratio. Consequently the greater v is, the less γ_3 and γ_4 deviate from each other.

Now for the computation of the quantity C. The greatest angular velocity of the aircraft arising in the curve may be designated by ϖ_{max} and the centrifugal moment by M_k. Should the screw even at the highest number of revolutions ω_{max} not incline more than 10 per cent. against its normal position we must have

$$C = M_{k\,max}/0.175 \quad . \quad . \quad . \quad . \quad . \quad (6)$$

whereby

$$M_{k\,max} = J_p \cdot \omega_{max} \cdot \varpi_{max}.$$

Hence the two screw frequencies are

$$\gamma_{3,4} = 3,4 \sqrt{(\omega_{max} \cdot \varpi_{max})} \sqrt{\{2/(v \pm 2)\}}.$$

The natural frequencies thus determined are consequently only dependent upon the size of the screw in so far as the value ω_{max} is determined therefrom.

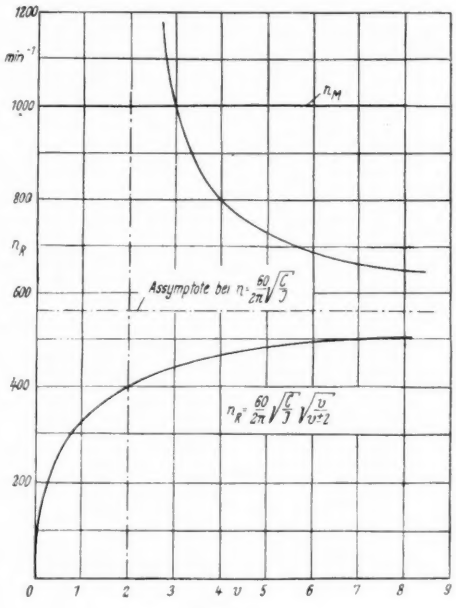

FIG. 3.

Resonance frequencies n_R of the elastically connected screw for various types of excitation $v = k_L$ or $i \cdot k_M$.

As an example let us choose a 1,500 h.p. engine with $i = 1.6$ and $\omega_{max} = 150\ s^{-1}$. Fig. 2 shows for this the dependency of the two natural frequencies n_3 and n_4 min.$^{-1}$ upon the size v for a value of $\varpi_{max} = 28^{-1}$. The natural frequencies n_M of the elastically suspended engine itself we can set somewhere in the neighbourhood of $n_M = 1{,}000$ min.$^{-1}$, so that all critical revolutions come far enough below the working revolution range.

$k_L = 3$ screw-side and $k_M = 1.5$ engine-side may be regarded as the lowest order of excitation; since we must have $v > 2$ it must be the upper of the two natural frequencies of the screw which generally excites. From Fig. 3 it is evident that the upper natural frequency for $v = 1.5 \cdot 1.6 = 2.4$ lies fairly high. The ratios being such that we must expect this type of excitation the dimension C must be reduced. Since, however, the screw would then incline during the curve even at values of $\varpi_{max} < 28^{-1}$ more than 10° deflections must be provided to prevent

this. For excitation orders > 3 screw-side and > 2 engine-side all critical revolutions, even with stronger springing C would come far enough under the range of working revolutions.

In conclusion yet a brief investigation of dimension C in respect of the coupling factor formed by it. By Eq. (6) we have with $J_p = 400$ kg./cms.^{2}, $\omega_{max} = 150\ s^{-1}$ and $\varpi_{max} = 28^{-1}$ a value of

$$C = 685{,}000 \text{ cm./kg.}$$

If $n_M = 1{,}000$ min.$^{-1}$ be true, *i.e.*,

$$\gamma_M = \sqrt{(c/\Theta)} = 105\ s^{-1}$$

there follows $\Theta = 1{,}300$ cm./kgs.^{2} and $c = 1{,}300 \cdot 11{,}000 = 14{,}300{,}000$ cm./kg.

$$K = \sqrt{\{C/(c+C)\}} = 0.214.$$

The uncoupling of engine and screw tipping motions is thereby accomplished and the correction for the above approximated calculation of natural frequency figures indicated.

V. Summary.

Various airscrew vibrational stresses are treated and the opposal thereof by elastic connection of screw and engine. In extension a measure was discussed which might protect the screw against tipping motions and thereby against the alternating centrifugal moments occasioned thereby. The corresponding vibration equations are appended and the dimensions of the springing to be adopted decided upon.

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A CONTRIBUTION TO CASCADE THEORY.

By J. Lockwood Taylor, D.Sc., *Associate Fellow.*

The theory of the flow past cascades or infinite series of aerofoils, regularly spaced, is of importance in connection with the design of turbo-blower blades, interference between propeller blade root sections, wind tunnel circuit design and kindred problems. The usual development of the theory involves some rather advanced applications of conformal representation which it is rather difficult to translate into numerical terms, and no general solution for an arbitrary form of aerofoil section is available. So far as lift interference is concerned, one of the main points on which information is required, approximate results of fairly general application can be obtained with the help of relatively simple mathematics, and the formulæ lend themselves readily to numerical work. The basic principle applied is that of the direct calculation of the downwash component of the flow due to the remaining aerofoils of the cascade, somewhat on the lines of Glauert's approximate biplane interference corrections.

(i) Uncambered Aerofoil at 0° Stagger.

Immediate application of the Glauert biplane correction:—

$$\alpha - \alpha_0 = (1/8\pi)\,(c/h)^2\,(C_L + 2C_{mo})$$

which assumes small chord-gap ratio (c/h) and which becomes

$$\alpha - \alpha_0 = (1/8\pi)\,(c/h)^2\,C_L$$

for zero camber ($C_{mo}=0$), for either wing of a biplane, or

$$\alpha - \alpha_0 = (1/4\pi)\,(c/h)^2\,C_L$$

for the middle of three aerofoils (triplane) giving

$$C_L/\alpha = 2\pi \div \{\,1 + \tfrac{1}{2}\,(c/h)^2\,\}$$

on putting $C_L = 2\pi\alpha_0$, leads to the correction factor

$$\{\,1 + \tfrac{1}{2}\,(c/h)^2 + \tfrac{1}{2}\,(c/2h)^2 + \ldots\,\}$$

for a cascade. The series adds up to

$$\{\,1 + (\pi^2/12)\,(c/h)^2\,\}$$

and the resulting numerical values of the lift slope are:—

c/h	$\frac{1}{2}$	1	2
C_L/α	5·21	3·45	1·465
Do. exact ...	5·24	3·67	2·0

The exact values of C_L/α, known from the full mathematical solution for a cascade of laminæ, show fair agreement up to the chord gap of about unity which is perhaps a better result than might have been expected in view of the assumptions. The particular assumption (c/h) small may easily be dropped; Glauert takes the downwash velocity proportional to x/h^2, where x is the abscissa from the centre of pressure, instead of the exact value $x/(h^2+x^2)$. Substituting the latter gives, for the cascade,

$$C_L/\alpha = 2\pi \div \left\{\,1 + \tfrac{1}{2}\left[\frac{c^2}{(1+c^2/4)} + \frac{(c^2/4)}{(1+c^2/16)} + \frac{(c^2/9)}{(1+c^2/144)} + \ldots\right]\right\}$$

and numerical evaluation of this series gives lift slopes in exact agreement with the correct theoretical values already quoted (c has been written here for c/h, *i.e.*, h taken as unity, which does not involve any loss of generality). Incidentally, the corrected approximate form of C_L/α for the biplane case greatly improves the agreement with the true values as quoted in Glauert, " Aerofoil and Airscrew Theory," 13-14.

(ii) Cambered Aerofoil, 0° Stagger.

It is readily verified that the downwash or interference effects just given are equivalent to calculating downwash velocity at the $\frac{3}{4}$ chord point and correcting the lift as if the corresponding change of incidence applied to the aerofoil as a whole (taking the effective downwash at $\frac{3}{4}$ chord includes the effect of downwash at mid-chord and of flow curvature, considered separately by Glauert). The C.P. is at $\frac{1}{4}$ chord for a symmetrical or straight line aerofoil (in the open) and the downwash is proportional to the separation between C.P. and $\frac{3}{4}$ chord. Thus the effect of cambering the centre line, which throws the C.P. farther back, can be estimated. An aerofoil working at about optimum C_L for its camber will have C.P. near .5 chord; the effective downwash is halved as compared with the above, and

$$C_L/\alpha = 2\pi \div \left\{ 1 + \tfrac{1}{4}\left[\frac{c^2}{(1+c^2/4)} + \frac{(c^2/4)}{(1+c^2/16)} + \ldots \right] \right\}$$

giving

c/h	$= \frac{1}{2}$	1	2
C_L/α	$= 5.70$	4.50	2.60

The loss of lift due to interference is less than without camber.

(iii) Constant Pressure Aerofoil, 0° Stagger.

It is to be supposed that a better approximation to the interference will be got by allowing for the distribution of the vorticity over the chord, instead of taking a concentration of vorticity at the C.P. A case which is fairly easy to calculate and which may not be far from the truth for C.P. 0.5 is that of constant tangential velocity or pressure on both upper and lower surfaces. Integration of the downwash or normal induced velocity at a point on one aerofoil, due to a uniform distribution of vorticity at the adjacent aerofoil, gives

$$\int_{-c}^{c} \frac{(x-x_1)\,dx_1}{h^2+(x-x_1)^2} = \tfrac{1}{2}\log\left[\frac{(h^2+x+c^2)}{(h^2+x-c^2)}\right]$$

For the whole cascade

$$\sum_{n=-\infty}^{n=\infty} \tfrac{1}{2}\log\left[\frac{(n^2h^2+x+c^2)}{(n^2h^2+x-c^2)}\right] = \log\frac{(\sinh \pi \,.\, x+c/h)}{(\sinh \pi \,.\, x-c/h)}$$

which replaces $\log[(x+c)/(x-c)]$ for a single constant pressure aerofoil of chord $2c$ in the open, *i.e.*, without interference. Accordingly the ratio of the camber line slopes, proportional to the normal velocity as just derived, with and without interference, to produce a given lift (at optimum incidence) is known. Integration of the respective slopes gives the ratio of the camber for a given lift, with and without interference, or the lift ratio for given camber.

$2c/h$	$\frac{1}{2}$	1	2
C_{L_0}/C_L	1.14	1.485	2.4
$2\pi \div (C_{L_0}/C_L)$ ...	5.5	4.23	2.62

The last line gives figures numerically comparable with those previously tabulated.

(iv) CONSTANT PRESSURE AEROFOIL WITH STAGGER.

Any of the results (i)-(iii) can obviously be extended to the more general case of a staggered series of aerofoils. Taking (iii) as the most nearly accurate and corresponding best with the practical case, generalisation gives

$$\log\left[\frac{\{\cosh[(2\pi/s)(x+c)\cos\sigma]-\cos[(2\pi/s)(x+c)\sin\sigma]\}}{\{\cosh[(2\pi/s)(x-c)\cos\sigma]-\cos[(2\pi/s)(x-c)\sin\sigma]\}}\right]$$

downwash in cascade, as compared with $\log(x+c/x-c)$ in the open, where s is the spacing of the aerofoils ($h=s\cos\sigma$ is the gap measured normally to the chord) $2c$ the chord, and σ the angle of stagger. The ratio C_L/C_{L_0} as defined above has been evaluated for the numerical case, $\sigma=45°$, $2c/s=1$, giving 1.118, or $2\pi\div(C_L/C_{L_0})=5.61$. The loss of lift is appreciably less than for zero stagger at the same solidity or chord-gap ratio, as in the known solution for staggered straight-line aerofoils, for which the corresponding figure is 5.3.

(v) GENERAL CAMBERED AEROFOIL.

The results (iii) and (iv) for the special case of uniform lift distribution over the chord can be generalised to cover any specified shape of lift distribution curve. Thus, summing the downwash at x due to an element of lift or vorticity at x_1,

$$\sum_{-\infty}^{\infty}\frac{(x-x_1)}{\{n^2h^2+(x-x_1)^2\}}=\frac{\pi}{h}\coth\frac{\pi\,.\,x-x_1}{h}$$

For a lift ordinate proportional to $f(x_1)$, integration gives as the total downwash at x

$$\frac{\pi}{h}\int_{-c}^{c}f(x_1)\coth\left\{\frac{\pi\,.\,(x-x_1)}{h}\right\}dx_1.$$

These expressions apply to the case of zero stagger, and the corresponding integral for the same lift distribution $f(x_1)$ on an isolated aerofoil, without cascade interference is

$$\int_{-c}^{c}\{f(x_1)\div(x-x_1)\}\,dx_1,$$

enabling the comparison of cambers required for a given lift to be made as before. Further generalisation for stagger angle σ and spacing s, gives as the appropriate integral for downwash or slope of camber line, corresponding with lift distribution $(f(x_1)$, in cascade

$$\frac{\pi}{s}\int_{-c}^{c}f(x_1)\frac{\{\cos\sigma\sinh[(2\pi/s)(x-x_1)\cos\sigma]+\sin\sigma\sin[(2\pi/s)(x-x_1)\sin\sigma]\}}{\{\cosh[(2\pi/s)(x-x_1)\cos\sigma]-\cos[(2\pi/s)(x-x_1)\sin\sigma]\}}\,dx_1$$

again to be compared with

$$\int_{-c}^{c}\{f(x_1)\div(x-x_1)\}\,dx_1$$

in the open.

It should be emphasised that all the results apply strictly to aerofoils of infinitesimal thickness and small camber at small angles of incidence, *i.e.*, they are simply extensions of the usual theory of thin aerofoils. Moderate thickness may be expected to alter the upper and lower surface velocities by similar amounts without serious change in lift distribution or total amount, and the restriction

to small camber is probably not very serious from a practical point of view. Second order effects such as the tangential induced velocity when the stagger is not zero have been ignored. Finite thickness at zero stagger can easily be included in the calculation, but the general case of finite thickness plus stagger presents difficulties. Two-dimensional flow, or infinite aspect ratio is of course assumed throughout.

REVIEWS.

CLOUD READING FOR PILOTS. By A. C. Douglas. John Murray. 1943. 10/-.

Glider pilots must surely be on friendly terms with the clouds, upon which the very essence of their favourite sport depends; so the authoress of the present book (better known to pre-war sporting flyers under her maiden name, Ann Edmonds) is particularly well qualified to write it. A foreword by Professor David Brunt gives authority to the book, although the authoress herself modestly disclaims any intention beyond that of giving some practical hints to pilots and putting them in a position " to find out more for themselves." The letterpress, describing cloud formations and the development of air masses and fronts, is clear and simple, but the real value of the work lies in its illustrations. Out of 120 pages, over 70 are taken up by cloud photographs arranged in orderly sequences, each with its date and an explanatory note. Seldom can so many exquisite pictures have been assembled into a single volume. In these days, when skimped and shoddy publications are all too common, this book, with its glossy paper, wide margins and artistic layout, is a joy to the eye as well as to the mind. It would make an ideal Christmas present, not only for pilots and prospective pilots, but for artists and nature-lovers as well.

METEOROLOGY SIMPLIFIED. By Joseph I. Fell. Sir Isaac Pitman and Sons. 1943. 1/-.

This new volume in the series, entitled " Pitman's Pocket Handbooks," covers the syllabus of the A.T.C. and I.T.W. and is written by an Air Navigation instructor. A plethora of books on elementary navigation has been written since the war, but most of these either ignore Meteorology altogether or dismiss it in a few casual pages. This primer is therefore likely to be of use to air cadets in the early stages of their training. A good deal of ground is covered in its 50 pages: The composition of the atmosphere; air temperature and pressure; convectional stability and instability; high and low pressure systems; general laws governing winds; local and regional winds (somewhat quaintly termed " Abnormal Winds "); types of cloud; warm and cold fronts, and a few remarks on ice accretion. There are frequent diagrams of the pictorial type popularised by certain American writers. When they are well done, these picture-diagrams are possibly easier to assimilate than the graph or diagram proper, but they are very liable to suggest the scribbles in a child's drawing-book. In the present case we cannot help feeling that an air cadet should be able to grasp the idea that it is night-time without seeing a crescent moon and a peppering of dots, no doubt intended to represent stars but equally suggestive of a snowstorm or a heavy dose of flak.

A HANDBOOK OF SEA NAVIGATION. By W. M. Smart. Longmans, Green and Co. 1943. 7/6.

Professor Smart is by now well established as an authority on the academical side of Navigation. This new book, written in the lucid and conscientious manner which we have come to expect from the author, takes its place in the series initiated by " An Introduction to Sea and Air Navigation." The second in the series, " Astronomical Navigation," carries the subject a step farther for the airman, while the present volume is written from the seaman's point of view. As might be expected, the two latter books have much in common; in fact the first 30 pages, dealing with geometrical principles and the geometry of the celestial

sphere, are virtually identical. From this point they diverge, " Sea Navigation " explaining in detail the Local Hour Angle of the Sun, Stars, Moon and Planets—which the airman, with less time at his disposal, takes ready-made from his *Air Almanac*—and describing the principal corrections to be made in taking sextant observations. The next chapter (The Astronomical Position Line) is again identical in its earlier portions, but diverges when it comes to the examples, which in the present volume are worked out by nautical methods. The concluding chapters treat of Position Lines in general; Sunrise and Sunset; Moonrise and Moonset; Stars and Planets. In conclusion, there is a thorough discussion of the Rhumb Line, accompanied by one of the best diagrams we have seen to illustrate the relationship between the Rhumb Line and the Great Circle (p. 131). The summary of formulæ in the Appendix gives various examples, such as the Half-Log Haversine Formula, that are not found in " Astronomical Navigation," while the specimen pages for reference are from the " Nautical Almanac " instead of the Air Almanac.

Now that the co-operation between the Sea and Air Forces is so close, we feel that air navigators could very profitably study this book, since the main difference between sea and air navigation is that the former is a more leisurely process, and can therefore be worked out in greater detail and to more precise limits of accuracy. An air navigator may not need to practise nautical methods himself, but his grasp of navigation will be all the better for knowing how they are carried out.

AIRCRAFT ELECTRICAL ENGINEERING.

By F. G. Spreadbury. Sir Isaac Pitman & Sons. 1943. 21/-.

This book is particularly welcome, in view of the small amount of literature on the subject. Such books as do exist are for the most part primers, assuming zero knowledge on the part of the reader. Mr. Spreadbury writes for men with an intermediate standard of mathematics and a general grasp of electricity, who wish to apply this knowledge to a specialised study of aircraft equipment.

He deals first with the theory of aero engine ignition, and then with the construction and testing of ignition apparatus; with aircraft generators and motors and the use of alternating current in aircraft. A very practical chapter explains the various systems used to counteract the effect of speed variations and maintain a constant voltage. Radio supply is handled from a purely theoretical point of view. The concluding chapters deal with ripple-smoothing circuits, magnets, and aircraft pyrometry, some highly condensed information on thermodynamics being included in this chapter. The book is illustrated by a number of excellent line-drawings, photographs and graphs, which no doubt accounts for its comparatively high price.

The electrical side of aircraft has not yet been properly visualized purely from the airman's point of view, nor standardized as fully as it should be, and the problems of weight of all electrical installations have yet to be thoroughly studied. There is no doubt in the future that aircraft will have more and more electrical installations of all kinds, and these must be designed from a different point of view from similar installations being used in other forms of engineering or commercial work.

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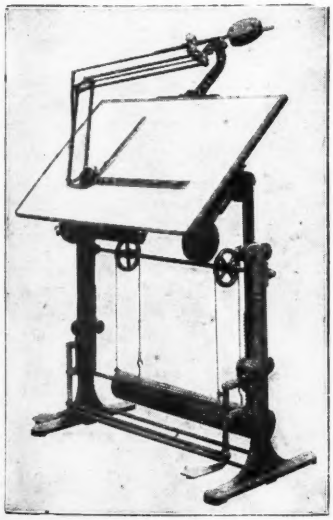

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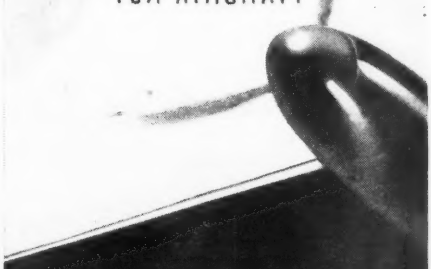

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INDEX TO ADVERTISERS

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Automatic Coil Winder & Electrical Equipment Co., Ltd.	xx
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Technological Institute of Gt. Britain	viii
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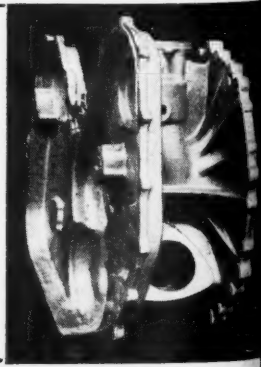

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